

MS 115a, Problem Set #7

assigned 11/21/07

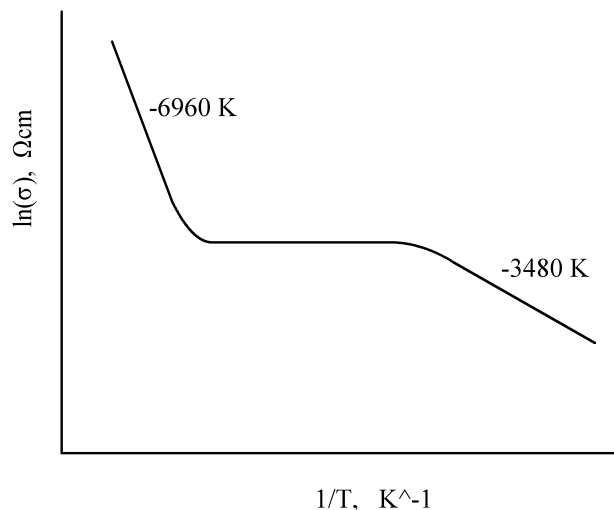
due 11/28/07

1. Consider the electrical properties of an ideal metal:
 - (a) Write down the functional dependence of electron density of states, $N(E)$, on energy. Plot this function.
 - (b) Write down the Fermi distribution function, $P_F(E)$. Plot this for a finite temperature.
 - (c) Write down the functional dependence of the electron distribution function, $F(E)$, on the electron density of states and the Fermi distribution function. Plot the electron distribution, as a function of energy, for a finite temperature (same temp. as part b).
2. Derive the density of states, $N(E)$, for (a) an electron in a 1-dimensional box and (b) an electron in a 2-dimensional square box. Assume the energy for the 2-D case to be given by

$$E_{n_x n_y} = \frac{\hbar^2 |k|^2}{2m} \quad \text{where } k = \frac{\pi}{L} (n_x \hat{k}_x + n_y \hat{k}_y)$$

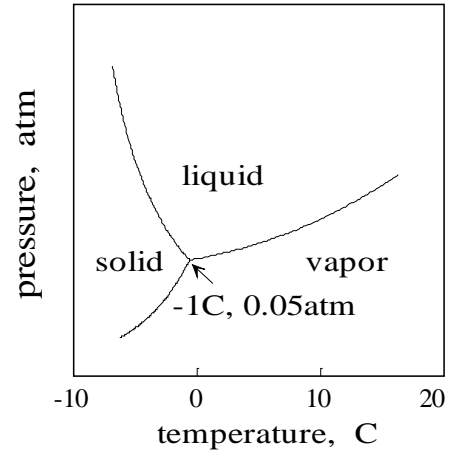
Compare these with the 3-D result (i.e., problem 1)

3. Explain why nonstoichiometric III-V compounds (where III implies Al, Ga or In and V implies P, As or Sb) are typically extrinsic semiconductors.
4. The electrical conductivity of a p-type semiconductor has been measured and behaves as shown below. The numbers on the plot indicate the slope of $\ln(\sigma)$ vs. $1/T$. Label the regions of this graph. Draw a schematic band diagram for this material in the form of E vs. $N(E)$. Indicate as many energy terms as you can (in units of eV).



5. The phase diagram of H₂O is shown to the right.

- a) For T₁ = -5°C plot schematically G_{solid}, G_{liquid} and G_{vapor} as functions of P.
- b) Do the same for T₂ = 10°C.
- c) What is $\left(\frac{\partial G}{\partial P}\right)_{T, \{N_i\}}$?

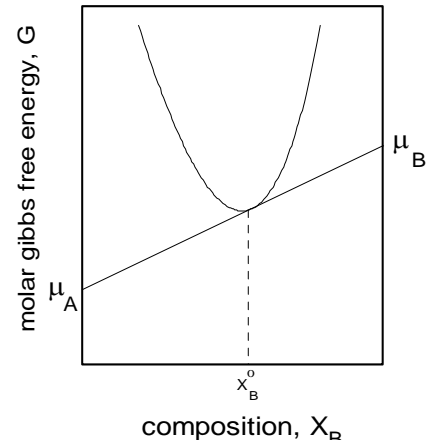


6. Show that the entropy (per mole) due to mixing of A and B atoms is given by

$$\Delta S^{\text{mix}} = -RX_A \ln X_A - RX_B \ln X_B.$$

- a) Begin with the statistical definition of entropy, $S = k_b \ln \Omega$,
- b) Compare the entropies of the mixed and unmixed solutions (having N_o atoms)
- c) Use Stirling's approximation, $\ln(N!) \sim N \ln(N)$ for large N, to evaluate S
- d) Note the relationship between k_b, R and N_o.
7. Show that for an ideal solution the slope of $d\Delta G^{\text{mix}}/dX_B \rightarrow -\text{infinity}$ as $X_B \rightarrow 0$, and approaches infinity as $X_B \rightarrow 1$. Interpret this result in terms of the concentration of impurities present in any material.

8. In class we noted the "Tangent Rule" for determining the chemical potential of a species in a solution from the Gibbs free energy curve as a function of composition. Specifically it was stated that the chemical potentials μ_A and μ_B are given by an extrapolation of a line, tangent to the Gibbs free energy curve at the composition of interest, to the compositions $X_B = 0$ and $X_B = 1$, as shown in the upper figure.



Derive this result by the following steps:

a) Show that the differential molar Gibbs free energy of a solution, at constant T and P is

$$dG = \mu_A dX_A + \mu_B dX_B$$

b) Knowing that $X_A + X_B = 1$, show that

$$\mu_B = G + X_A(dG/dX_B)$$

c) Use the lower figure to demonstrate the equivalence of the tangent rule and the equation of step (b).

