MS 115a, Problem Set #7

assigned 11/14/12
due 11/21/12

1. Consider the Bohr model of an atom.

   (a) Show that the velocity of an electron orbiting a nucleus is given by

   $v = Ze^2/4\pi\varepsilon_o n\hbar$

   (b) Find the time period for one revolution.

2. Using the Bohr model of hydrogen calculate the energy of the photon emitted when an electron jumps from the n = 3 to the n = 1 state.

3. Suppose a photon of wavelength 0.09 Å is absorbed by an electron in potassium with principle quantum number, n, equal to 1. Some of this energy is used to remove the electron from the atom and the remainder is stored as kinetic energy. Find the velocity of the electron.

4. Write down the $\psi_{100}(r,\theta,\varphi)$ electron orbital for a single electron atom. Use this to generate electron density plots for the bonding and anti-bonding molecular orbitals of the H_2^+ molecule. That is, plot the electron probability densities for these two orbitals (or states) as a function of position for an H_2^+ molecule with a fixed interatomic distance.

5. Consider a particle in a 1-dimensional box of length a and centered about x = 0. The potential well is given by:

   $V(x) = 0$ at $|x| < a/2$
   $V(x) = \infty$ at $|x| \geq a/2$.

   Beginning with the time independent Schroedinger equation, derive the wave function(s) and energy of the particle. Note the boundary conditions:

   $\psi(x) = 0$ at $|x| \geq a/2$, and $\int_{-a/2}^{a/2} \psi^*(x) \cdot \psi(x) dx = 1$.

6. For the particle of problem (4), evaluate the expectation values of x, p, and x^2.

7. Consider electrons in an ideal metal:

   (a) Write down the functional dependence of the electron density of states, $N(E)$, on energy. Plot this function.

   (b) Write down the Fermi distribution function, $P_F(E)$. Plot this for a finite temperature.

   (c) Write down the functional dependence of the electron distribution function, $F(E)$, on the electron density of states and the Fermi distribution function. Plot the electron distribution, as a function of energy, for a finite temperature (same temp. as part b).

8. Consider the Fermi distribution function, $P_F(E)$. (a) Show that at T = 0, $P_F(E) = 1$ for all energy states $E < \varepsilon_F$ and $P_F(E) = 0$ for all energy states in which $E > \varepsilon_F$. (b) Show that $P_F(E) = ½$ for $E = \varepsilon_F$ (at any temperature).

9. Derive the density of states, $N(E)$, for (a) an electron in a 1-dimensional box and (b) an electron in a 2-dimensional square box. Assume the energy for the 2-D case to be given by

$$E_{n_x n_y} = \frac{\hbar^2 |k|^2}{2m} \quad \text{where } k = \frac{\pi}{L}\left(n_x \hat{k}_x + n_y \hat{k}_y\right)$$

Compare these with the 3-D result in a plot of N(E) vs. E.

10. The Fermi energy of potassium is 2.12 eV (at T = 0). (a) Calculate the number of conduction electrons per unit volume in potassium. (b) Compare this to the value implied by the crystal structure, BCC with a = 5.328 Å.

11. Explain why the electrical resistivity of a metal quenched from high temperature is greater than the resistivity of one that has been slowly cooled.