MS 115a, Problem Set #9

assigned 11/28/12
due **12/6/12**
will **not** be accepted late

1. Show that the chemical potential of species A in an ideal solid solution is given by

   $\mu_A = \mu_A^o + RT\ln(X_i)$ where μ_A^o is the chemical potential in the pure state.

2. The enthalpy of mixing of a regular solution is given by $\Delta H^{mix} = \Omega X_A X_B$, where Ω is a constant that can be related to the energy parameter, ϵ, of the quasichemical model of solutions. Show that for $\Omega > O$ (clustering) the maximum temperature at which phase separation occurs is given by $T_{max} = \Omega/2R$. Hints: (1) note that the maximum occurs at $X_A = X_B = 0.5$; (2) note that phase separation occurs because ΔG^{mix} has a double minima shape at low temperature, thus it changes from positive curvature to negative curvature at T_{max}.

3. The metallic elements A and D form a solid solution at room temperature. On cooling, phase separation occurs and the critical temperature, T_{max}, is 265K. What is the difference between the energy of A-D bonds and the average of the A-A and D-D bonds?

Problems 4-10 refer to the figure below and to the right. You will need to copy it onto your homework paper.

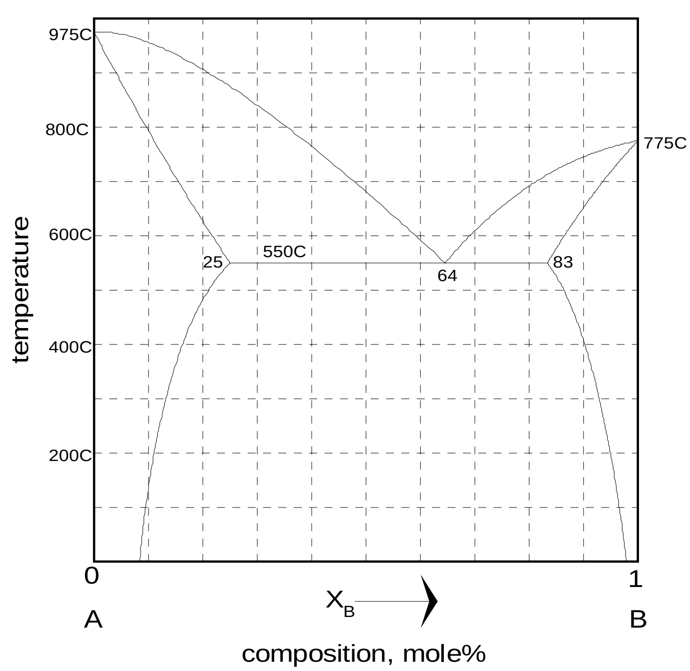
4. The melt temperatures of pure A and pure B are 975 and 775°C, respectively. Label these points as well as the regions and boundaries on the phase diagram.

5. What is the name given to this type of phase diagram?

6. If pure A has the hexagonal close-packed structure, what is the structure of pure B?

7. What is the composition of each phase (at equilibrium) at 450°C for an alloy with 35 mol% B? What is the phase fraction of each phase?

8. What is the maximum solubility of B into A? At what temperature is this maximum attained? What is the maximum solubility of A into B? At what temperature is this maximum attained?



9. Draw the Gibbs free energy curves of the liquid and the solid as a function of composition (X_B), for the following temperatures:

   $T_1 > 975°C$ $T_2 = 975°C$ $975°C > T_3 > 775°C$

$T_4 = 775°C$ $775°C > T_5 > 550°C$ $T_6 = 550°C$

$T_7 < 550°C$

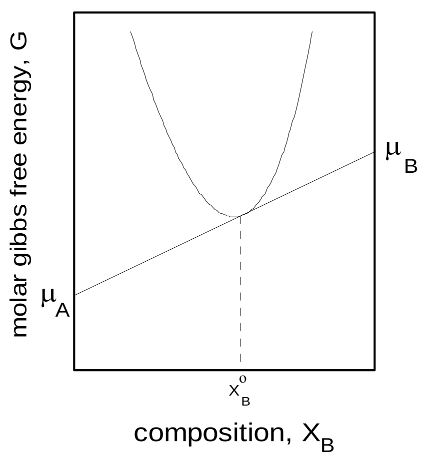
10. Using Gibbs free energy curves at 850°C (similar to $T = T_3$ of problem 9):

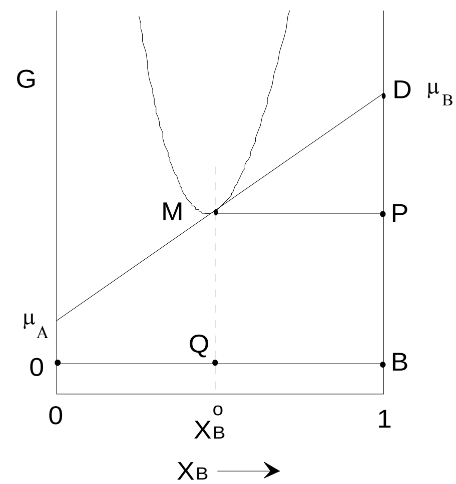
  a) Show the chemical potentials of components A and B for an alloy with 20 mol% B.

  b) Show that in the two-phase region the chemical potential of A in the solid phase is equal to that in the liquid phase

  c) Plot μ_A as a function of X_B. Pay close attention to the behavior of μ_A within the two-phase region.

10. In class we noted the "Tangent Rule" for determining the chemical potential of a species in a solution from the Gibbs free energy curve as a function of composition. Specifically it was stated that the chemical potentials μ_A and μ_B are given by an extrapolation of a line, tangent to the Gibbs free energy curve at the composition of interest, to the compositions $X_B = 0$ and $X_B = 1$, as shown in the left-side figure.





Derive this result by the following steps:

  a) Show that the differential molar Gibbs free energy of a two-component solution, at constant T and P is

$$dG = \mu_A dX_A + \mu_B dX_B$$

  b) Knowing that $X_A + X_B = 1$, show that

$$\mu_B = G + X_A(dG/dX_B)$$

  c) Use the right-side figure to demonstrate the equivalence of the tangent rule and the equation of step (b).

11. Show that for an ideal solution the slope of $d\Delta G^{mix}/dX_B \rightarrow$ -infinity as $X_B \rightarrow 0$, and approaches infinity as $X_B \rightarrow 1$. Interpret this result in terms of the concentration of impurities present in any material.