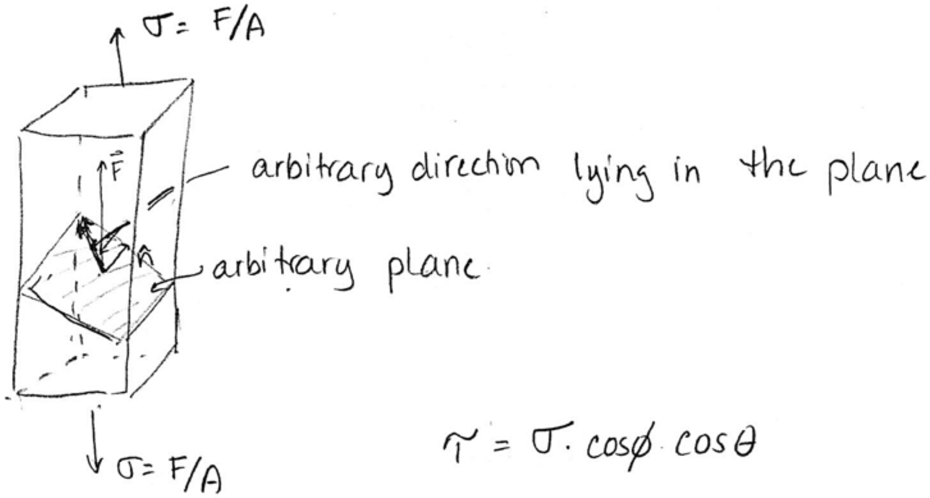
Resolved shear stress.



$$\tau = \sigma \cdot \cos\phi \cdot \cos\theta$$

$\phi = \angle$ $\hat{n}$ to plane and $\vec{F}$

$\theta = \angle$ direction in plane and $\vec{F}$

<u>example</u>: ccp metal, single crystal is pulled along [100].

slip system: (111) and [110].

ϕ: between $\hat{n}_{(111)}$ and $\hat{n}_{[100]}$.

cubic: $\hat{n}_{(111)} = \hat{n}_{[111]}$ direction: [111]

unit vector: $\frac{1}{\sqrt{3}}\hat{a}_1, \frac{1}{\sqrt{3}}\hat{a}_2 \ \frac{1}{\sqrt{3}}\hat{a}_3$

$\hat{n}_{[100]} = \hat{a}_1, 0, 0$

$\hat{n}_1 \cdot \hat{n}_2 = |n_1| \cdot |n_2| \cdot \cos\varphi$

here: $\left(\frac{1}{\sqrt{3}} + 0 + 0\right) = 1 \cdot 1 \cdot \cos\phi = \frac{1}{\sqrt{3}}$

θ between $\hat{n}_{[110]}$ and $\hat{n}_{[100]}$ $\quad \hat{n}_{[110]} = \frac{1}{\sqrt{2}}\hat{a}_1, \frac{1}{\sqrt{2}}\hat{a}_2, 0$

$\hat{n}_1 \cdot \hat{n}_2 = \left(\frac{1}{\sqrt{2}} + 0 + 0\right) = \cos\phi = \frac{1}{\sqrt{2}} \Rightarrow \tau_{res} \cdot \frac{\sigma}{\sqrt{6}}$

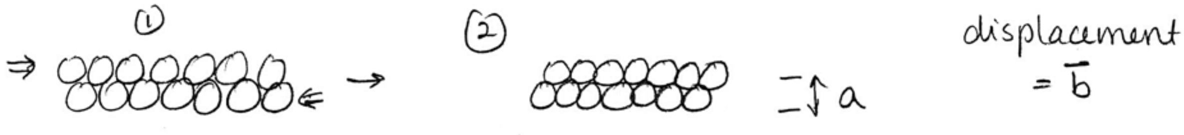
this is the shear stress the slip system feels, will disl. move?

The resolved shear stress that is large enough to activate a slip system → critical resolved shear stress.

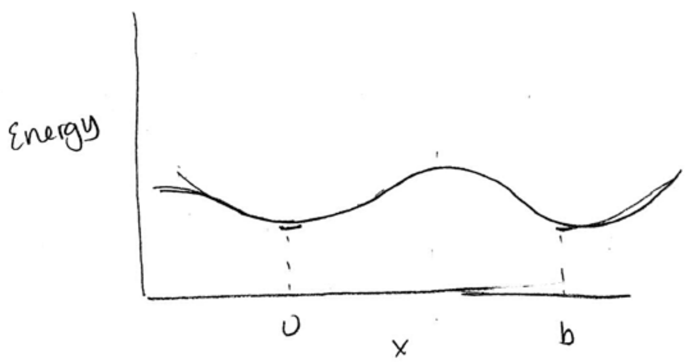
τ_{crss} → typically measured & tabulated for different materials (metals).

Compare the theoretical max value if shear occurred w/out dislocation glide.

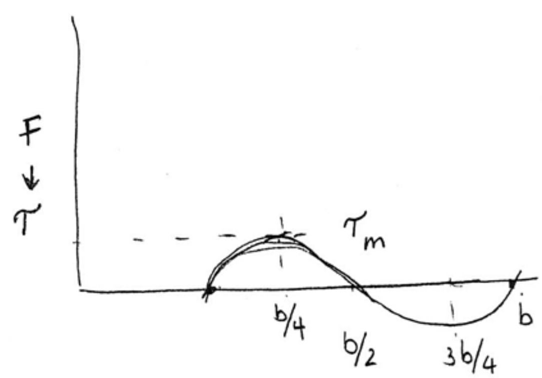
Theoretical Shear Strength



how much force is required ⇒ how much shear stress?



$F = \frac{dE}{dx}$



$\tau \propto F_{shear}$

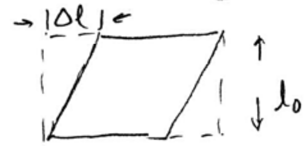
$\tau = F/A$

assume sinusoidal.

want τ_m.

$\tau = \tau_m \cdot \sin\left(\frac{2\pi x}{b}\right)$ - shear stress need for moving atoms

by definition $\tau = G \cdot \gamma$ $\quad \gamma = \frac{\Delta l}{l_0}$



for atoms sliding past, $\gamma = \frac{\Delta l}{l_0} = \frac{x}{a}$

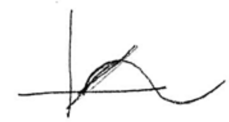
$\Rightarrow \tau = G \cdot \gamma = G \cdot \frac{x}{a}$

for small τ, $\tau \approx \tau_m \left(\frac{2\pi x}{b} \right) = G \cdot \left(\frac{x}{a} \right)$

(essentially estimating τ_m from slope @ $\tau = 0$)

$$\Rightarrow \tau_m = \frac{G \cdot b}{2\pi a} \simeq \frac{G}{2\pi}$$

alternatively, what is τ at $x = b/4$



(essentially estimating τ_m assuming constant slope)

$\gamma = \frac{b}{4a}$ $\quad \tau = G \left(\frac{b}{4a} \right) \approx \frac{G}{4}$

$\tau_{max} = \frac{\sigma}{2} \Rightarrow \sigma_y = 2 \cdot \tau_{max} \approx \frac{G}{2}$ or $\frac{G}{\pi}$

we would expect yield strength to be this high if atoms in a plane simultaneously slid past one another.

Actual value $<< G \Rightarrow$ lead to hypothesis of dislocations.

Strategies for strengthening metals

$\equiv$ increasing σ_y

- work hardening $\Delta\sigma_y \propto \sqrt{\rho}$ dislocation # density

  opposite: anneal to soften.

- Solute hardening (point defects)

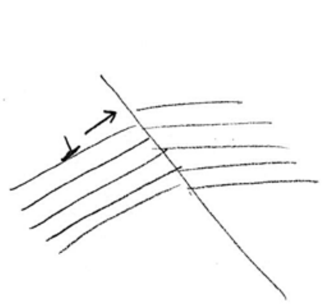
- precipitate hardening

  coherent: $\Delta\sigma_y \propto \epsilon \cdot f_v$ (lattice strain field: ϵ; volume fraction precipitates: f_v)

  incoherent
  $\Delta\sigma_y \propto 1/\lambda$ ← interparticle spacing

  for a given volume fraction small, more numerous ppts are more effective.

- grain boundary hardening (planar defects)



$$\sigma_y = \sigma_0 + k d^{-1/2}$$

Hall-Petch relation.

boundaries present barriers to $\perp$ motion.