$$\psi(r,\theta,\varphi) = \underset{\substack{\Downarrow\\ n}}{R(r)}\cdot\underset{\substack{\Downarrow\\ \ell}}{\Theta(\theta)}\cdot\underset{\substack{\Downarrow\\ m}}{\Phi(\varphi)}$$

quantum #

each is a known analytical function.

relationship to chemical notation

$n \rightarrow$ K, L, M "shells" $n = 1, 2, 3 \ldots$ → energy.

$\ell \rightarrow$ s, p, d "orbitals" $\ell = 0, 1, 2, \ldots n-1$

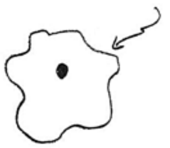
$m \rightleftarrows p_x, p_y, p_z$ "orientation" $m = -\ell, -\ell+1, \ldots 0, \ldots \ell-1, \ell$

} angular momentum.

$s = \frac{1}{2}$ or $-\frac{1}{2}$ additional spin quantum number (up or down).

Physical interpretation of quantum # n, Bohr atom

assumption: electron orbits nucleus. → restricts r



such that $n\lambda = 2\pi r_n$ $\Rightarrow = \frac{n\hbar}{p} = 2\pi r_n$

recall $\lambda = h/p$

$$\Rightarrow r_n = \frac{n\hbar}{p} = \frac{n\hbar}{mv}$$

Balance forces. $(F = \frac{dV}{dr})$

electrostatic centrifugal

$$\frac{Ze^2}{4\pi\epsilon_0 r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}} \Rightarrow r = \frac{Ze^2}{4\pi\epsilon_0 \cancel{m}v^{\cancel{2}}} = \frac{n\hbar}{\cancel{m}v}$$

$$\Rightarrow v_n = \frac{Ze^2}{4\pi\epsilon_0 \cdot n\hbar}$$

velocity ↗

$$r_n = \frac{n\hbar}{mv} = \underbrace{\boxed{4\pi\epsilon_0 \frac{(n\hbar)^2}{mZe^2}}}_{\text{Bohr radius}}$$

for hydrogen:

$r_{\substack{n=1\\ Z=1}} = 0.529\,\text{Å} = a_0$

What can we say about the energy?

$$E_{tot} = E_k + E_p$$

$$= \frac{1}{2}mv^2 + \frac{-Ze^2}{4\pi\epsilon_0 r}$$

want $E_{tot} = f(r)$. eliminate v.

$\Rightarrow E_{tot} = f(n)$.

from balance of forces.

$$mv^2 = \frac{Ze^2}{4\pi\epsilon_0 r}$$

$$\Rightarrow E_{tot} = \frac{1}{2}\left(\frac{Ze^2}{4\pi\epsilon_0 r}\right) - \frac{Ze^2}{4\pi\epsilon_0 r}$$

$$= -\frac{1}{2}\frac{Ze^2}{4\pi\epsilon_0 r}$$

know $r_n = 4\pi\epsilon_0 \frac{(n\hbar)^2}{mZe^2}$

$$\Rightarrow E_{tot} = -\frac{1}{2}\frac{mZ^2e^4}{(4\pi\epsilon_0)^2(n\hbar)^2} = \frac{-13.6\,Z^2}{n^2}\,\text{eV}$$

$Z=1$ $n=1$
-13.6 eV $= 1$ Ry

- Same result obtained from proper Q.M.
- Energy to change state (or to remove an electron)
- Degenerate w.r.t ℓ, m_ℓ, s

- - - -

Electron state characterized by 4 QNs: n, ℓ, m_ℓ, s.

simplest. $$\underbrace{\psi_{\underset{n\ \ \ell\ \ m_\ell}{100}}}_{\text{1s orbital}}(r,\theta,\varphi) = \underset{\substack{\uparrow \\ \text{known constant.}}}{A}\, e^{-Zr/a_0}$$