Density of States

\[ N(E) \sim E^{1/2} \]

Distribution of electrons within available states.

Electrons obey Pauli exclusion principle: Fermions.

Probability that an energy level is occupied, \( P(E) \).

Fermi distribution function

\[ P_f(E) = \begin{cases} 1 & E \leq E_F \\ 0 & E > E_F \end{cases} \]

0 Kelvin sequentially states are occupied until no more electrons to fill them defined as \( E_F \) Fermi energy.

Free electron metal: \( E_F \) simply fixed by \# of electrons available to fill the states.

\[ \# \text{ electrons} = N_0 = N_{qs}(E_F) \times 2 \pi^2 \text{ spin} \]

\[ \# \text{ states up to energy } E_F \]

\[ N_0 = \frac{L^3}{3} \left( \frac{8m}{\hbar^2} \right)^{3/2} E_F^{3/2} \]

\[ E_F = \frac{\hbar^2}{8m} \left( \frac{3}{\pi} \left( \frac{N_0}{L^3} \right) \right)^{2/3} \]

\( \eta \equiv \# \text{ free electrons/unit volume} \)

\( \varepsilon_g \text{ Ca metal } \Rightarrow \text{ Ca}^{2+} \Rightarrow 2 \text{ free electrons/atom} \)

from density of structure: atoms/volume
for \( T > 0, k \) use formal definition of \( P_F(E) \)

\[
P_F(E) = \frac{1}{\exp\left(\frac{E - E_F}{k_b T}\right) + 1}
\]

thermal excitation to higher energy states.

\[ P_F(E_F) = 0.5 \text{ at all } T \]

In which energy states are the electrons?

Electron distribution function: \( F(E) = 2N(E) \cdot P_F(E) \)
Now, consider atom cores.

Recall: $\lambda_n = \frac{2L}{n}$

$$\psi_n = \left\{ \begin{array}{ll}
\frac{1}{L} \sin \left( \frac{n\pi x}{L} \right) & \text{even} \\
\frac{1}{L} \cos \left( \frac{n\pi x}{L} \right) & \text{odd}
\end{array} \right.$$

- $\lambda = \frac{2a}{n}$
  \( (x = \frac{n\pi}{\lambda} = \frac{n\pi}{a}) \)

$\rightarrow$ discontinuity in $N(E)$

$\rightarrow$ forbidden energies $\Rightarrow$ bands

$\rightarrow$ electrons are 'scattered' by atoms

Extend $E$ to higher $E$ like $E(k)$.

$\rightarrow$ Fill states up to $E_f$ $\Rightarrow$ 3 possibilities (at $0, K$)

N(E)

- $E$ and $E_f$
  - solid
  - free electron

$E = \frac{k^2}{2m}$

- Metal
- Semiconductor/Insulator
- Metal
  (alkaline earth metal)
Electrical Conductivity

- apply \( V \), measure \( I \).

Ohm's Law: \( I = \frac{V}{R} \) or resistance

Definitions:
- \( \rho = \frac{RA}{L} = \frac{1}{\sigma} \) resistance
- \( E = \frac{V}{L} \) conductivity
- \( J = \frac{I}{A} \) field
- \( \text{current density} \)

Rewrite Ohm's Law: \( J = \eta \cdot E \)

\( \eta \) want to evaluate charge/area time

By definition: \( J = \eta \cdot e \cdot V_d \)

\( \frac{N}{L^3} \) charge \( \frac{\text{vol}}{\text{area} \times \text{time}} \)

\( n \) charge density

if we have how \( V_d \) responds to \( E \) → have \( \sigma \).

Free electron gas model

- apply \( E \Rightarrow F = meE = ma \)

\( F = ma \Rightarrow a = \frac{meE}{m} \)

Electron should accelerate infinitely...

→ collisions

\( V_d = a \cdot T = \frac{meE}{m} \cdot T \)

\( \Rightarrow \eta = \frac{e^2}{m} \cdot \frac{T}{E} \rightarrow \sigma \)

\( \bar{\ell} = \text{dist between collisions} \Rightarrow V_d = \frac{\bar{\ell}}{\eta} \)

\( \Rightarrow \sigma = \eta \frac{e^2}{m} \cdot \bar{\ell} = \eta \frac{e^2}{m} \frac{I}{V_d} \)
turns out - only electrons w/ energy close to $E_F$ participate

$$\Rightarrow U_d \rightarrow V_F \quad \text{velocity of electron at Fermi level}$$

\[
\text{classical: } E = \frac{1}{2} m v^2 \Rightarrow v = \left( \frac{2E}{m} \right)^{\frac{1}{2}}
\]

\[
\text{insert } E = E_F = \frac{\hbar^2}{8m} \left( \frac{3}{\pi} \eta \right)^{\frac{2}{3}}
\]

$$\Rightarrow V_F = \frac{\hbar}{2m} \left( \frac{3\eta}{\pi} \right)^{\frac{1}{3}}$$

more definitions

\[
J = J E = \eta \mid e \mid U_d
\]

where $\eta = \frac{J}{E} = \eta \mid e \mid \frac{U_d}{E} = \eta \mid e \mid \mu \quad \text{mobility}$

metals $\eta \sim$ constant electrons in the 'free electron gas'

$Ca \rightarrow Ca^{2+} + 2e^-$, also $E_F$

$\mu \sim \bar{l}$ mean free path between collisions

contributions to collisions: lattice (thermal) vibrations, impurities, grain boundaries.

\[
\frac{1}{\bar{l}_{tot}} = \frac{1}{\bar{l}_c} \Rightarrow P_{tot} = \frac{\xi}{\xi} P_c
\]

$P_{th}: \bar{l}_{th} = \frac{C}{T} \Rightarrow P_{th} \propto T$

$P_{imp}: \bar{l}_{imp} = \frac{C}{\chi_{imp}} \Rightarrow P_{imp} \propto \chi_{imp}$

$P \propto x_{Ni} (1 - x_{Ni})$