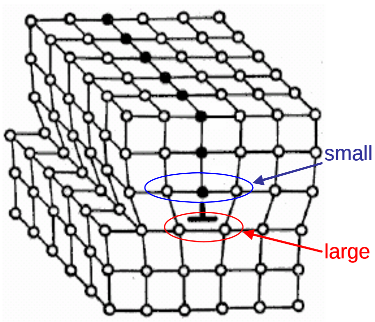
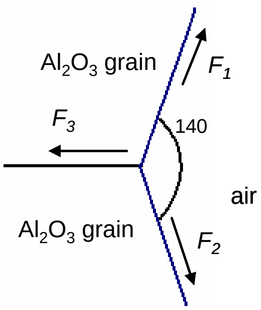
MS115A HW5 Solution set total 44 points

1. (5) Substitutional solute atoms introduce stress into the crystalline structure of the host (or matrix) atom. The sign of this stress (compression or tension) depends on the relative sizes of the solute and host atoms. Some of this stress can be relieved if the substitutional atom is located close to an edge dislocation.
(a) (3) Draw a schematic picture of an edge dislocation, and indicate the positions around the dislocation where large and small substitutional atoms would be found.
(b) (2) In which class of materials do you expect dislocations to be more mobile, high purity materials or ones with significant concentrations of solute atoms?



(a)
(b) Substitutional atoms pin dislocations because the distortion by a dislocation is moderated by the distortion caused by the size difference between matrix and substitutional atoms. So, dislocations are more mobile in high purity materials.

2. (3) Al_2O_3 has a surface energy of 905 erg/cm2 (with respect to air). After a thermal etch, it is observed that the pits formed at grain boundaries have an interior angle of 140º. What is the grain boundary surface energy?



F_1, F_2 and F_3 balance with each others. So, F_1, F_2 and F_3 satisfy that

$$F_3 = (F_1 + F_2) \cos 70$$

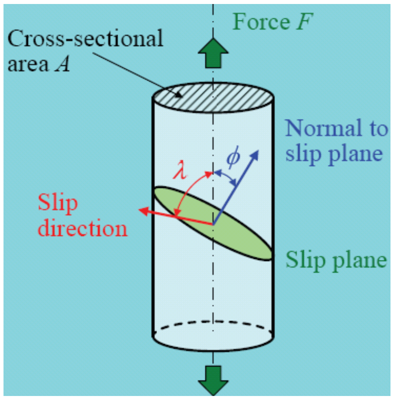
now,

$$E_{g.b.} \sim F_3 \text{ and } F_1 = F_2 \sim E_{air}$$

Therefore,

$$E_{g.b} = 2*905*\cos 70 = 619 \text{ erg/cm}^2$$

3. (6) Suppose you have a single crystal of a cubic close-packed metal which is known to have a critical resolved shear stress of 55.2 MPa.
(a) (3) Find the largest normal stress that could be applied to a bar of this material in the [1 1 2] direction before dislocations begin to move in the [-1 0 1] direction in the (1 1 1) plane.
(b) (3) Now let this be a body centered cubic metal with slip system [1 1 1](-1 0 1). Find the largest normal stress that could be applied in the [1 1 2] direction.



normal stress; $\sigma = \frac{F}{A}$ shear stress; $\tau = \frac{F}{A}\cos\phi\cos\lambda$

(a) $\cos\lambda$ = [112][-101]/|112||-101| = $1/(6*2)^{1/2}$
$\cos\phi$ = [112][111]/|112||111| = $4/(6*3)^{1/2}$
$\sigma = \tau/(\cos\phi*\cos\lambda)$ = 203 MPa

(b) $\cos\lambda$ = [112][111]/|112||111| = $4/(6*3)^{1/2}$
$\cos\phi$ = [112][-101]/|112||-101| = $1/(6*2)^{1/2}$
$\sigma = \tau/(\cos\phi*\cos\lambda)$ = 203 MPa

4. (5) (a) (3) Calculate the critical resolved shear stress in a crystal if a stress of 170 MPa in the [1 0 0] direction is required to move a dislocation in the [1 1 -1] direction on the (1 0 1) plane.
(b) (2) Is this a BCC or CCP metal? Why?

(a) $\cos\lambda$ = [100][11-1]/|100||11-1| = $1/(1*3)^{1/2}$

$\cos\phi$ = [100][101]/|100||101| = $1/(1*2)^{1/2}$

$\tau = \sigma*\cos\phi*\cos\lambda$ = 69.4 MPa

(b) This is BCC. Because the slip system is <1-11> {110} (**The slip system is <1-10> {111} for CCP)

5. (4) The yield stress of mild steel is 207 MPa. A specimen has a diameter of 0.01 m and a length of 0.10 m. It is loaded in tension to 1000 N and deflects 6.077×10^{-6} m.
(a) (2) Compute whether the stress is above or below the yield stress.
(b) (2) If the stress is less than the yield stress, calculate Young's modulus.

(a) $\sigma = F/A = 1000/(\pi*0.005^2)$ = 12.7 MPa < 207 MPa It's below the yield stress.

(b) $E = 12.7\times10^6/(6.077\times10^{-6}/0.10) = 2.09\times10^{11}$ Pa = 209 GPa

6. (6) Young's moduli for Al, Cu and W are 70,460, 122,500 and 388,089 MPa, respectively. Assuming the materials do not yield, compute the deflections in specimens of each material when subjected to a load of 5000 N. The specimens are 1.00 m long with a cross section of 1 cm $\times$ 1 cm.

$\Delta l = \varepsilon*l = l*\sigma/E$

Al; $1.00*(5000/0.01^2)/(70460*10^6) = 7.10*10^{-4}$m

Cu; $1.00*(5000/0.01^2)/(122500*10^6) = 4.08*10^{-4}$ m

W; $1.00*(5000/0.01^2)/(388089*10^6) =1.29*10^{-4}$ m

7. (2) The strength of aluminum oxide (alumina) can be as high as 4000 MPa and the fracture toughness can be as low as 2.5 MPa $\sqrt{\text{m}}$. A sample of alumina contains flaws that are 100 μm or less in size. Estimate the stress at which fracture will occur in this specimen.

$\sigma_{fracture} = K_c/(\pi c)^{1/2} = 2.5/(3.14\times50\times10^{-6})^{1/2}$ = 200 (MPa)

or

$\sigma_{fracture} = K_c/(c)^{1/2} = 2.5/(50\times10^{-6})^{1/2}$ = 353 (MPa)

8. (6) Consider the Bohr model of an atom.
(a) (3) Show that the velocity of an electron orbiting a nucleus is given $v = \dfrac{Ze^2}{4\pi\varepsilon_0 n\hbar}$
(b) (3) Find the time period for one revolution.

(a)

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \qquad (1)$$

$$\lambda = \frac{h}{mv} \qquad (2)$$

$$2\pi r = n\lambda \qquad (3)$$

from (2) and (3)

$$r = \frac{n\hbar}{mv}$$

plug in it to (1) and get

$$v = \frac{Ze^2}{4\pi\varepsilon_0 n\hbar}$$

(b) $$\omega = \frac{2\pi r}{v} = \frac{2\pi n\hbar}{mv^2} = \frac{2\pi\hbar n(4\pi\varepsilon_0 n\hbar)^2}{m(Ze^2)^2} = \frac{32\pi^3 n^3\hbar^3\varepsilon_0^{\ 2}}{mZ^2e^4} = \frac{4n^3h^3\varepsilon_0^{\ 2}}{mZ^2e^4}$$

9. (3) Using the Bohr model of hydrogen calculate the energy of the photon emitted when an electron jumps from the n = 2 to the n = 1 state.

$$E_n = \frac{mv^2}{2} - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{mv^2}{2} = \frac{-mZ^2e^4}{32\pi^2\varepsilon_0{}^2 n^2\hbar^2}$$

$$\Delta E = E_2 - E_1 = \frac{-mZ^2e^4}{16\pi^2\varepsilon_0{}^2\hbar^2}\left(\frac{1}{4} - 1\right) = \frac{3mZ^2e^4}{128\pi^2\varepsilon_0{}^2\hbar^2}$$

$$\Delta E = \frac{3\cdot(9.109e-31)\cdot(1.602e-19)^4}{128\cdot\pi^2\cdot(8.854e-12)^2\cdot(1.054e-34)^2} = 1.64\cdot 10^{-18} \quad (J)$$

$$= 10.2\ eV$$

or

$$\Delta E = -\frac{13.6Z^2}{n_1^2} - -\frac{13.6Z^2}{n_2^2} = \frac{3}{4}\times 13.6 = 10.2 \quad \text{(eV)}$$

10. (4) Suppose a photon of wavelength 0.09 Å is absorbed by electron in potassium with principle quantum number, n, equal to 1. Some of this energy is used to remove the electron from the atom and the remainder is stored as kinetic energy. Find the velocity of the electron.

$$E = E_{kin} + E_{remove} = \frac{mv^2}{2} + \frac{Ze^2}{4\pi\varepsilon_0 r} = \frac{hc}{\lambda}$$

$$v = \sqrt{\frac{2hc}{\lambda m} - \frac{Ze^2}{2m\pi\varepsilon_0 r}} = \sqrt{\frac{2hc}{\lambda m} - \frac{Z^2e^4}{8\pi^2\varepsilon_0{}^2 n^2\hbar^2}}$$

$n = 1,\ Z = 19,\ \lambda = 0.09e-10$

$$v = \sqrt{\frac{2\times 6.626\times 10^{-34}\times 2.998\times 10^8}{0.09\times 10^{-10}\times 9.109\times 10^{-31}} - \frac{19^2\times(1.602\times 10^{-19})^4}{2\times(8.854\times 10^{-12})^2\times(6.626\times 10^{-34})^2}}$$

$$= 2.12\times 10^8 (m/s)$$

Of course you can use following equation instead,

$E_{remove} = 13.6\times Z^2$ (eV)

So,

$$E_{kin} = \frac{mv^2}{2} = \frac{hc}{\lambda} + 13.6eZ^2$$

$$v = \sqrt{\frac{2hc}{\lambda m} - 2\times 13.6Z^2 em} = \sqrt{\frac{2\times 6.626\times 10^{-34}\times 2.998\times 10^8}{0.09\times 10^{-10}\times 9.109\times 10^{-31}} - \frac{2\times 13.6\times 19^2\times 1.602\times 10^{-19}}{9.109\times 10^{-31}}}$$

$$= 2.16\times 10^8 (m/s)$$