1. Using the LCAO model for insulating solids (where insulators means all non-metallic solids) compare the effective mass of more ionic compounds from those that are more covalent.

2. Consider the Kronig-Penney model for a 1-dimensional solid with a periodic potential, where the height and width of the barrier are arbitrary. Using the requirements that the wave function and its first derivative be single-valued and continuous, derive four equations that place restrictions on the allowed values of $k$, $\alpha$, and $\beta$, and therefore $E$.

3. The Kronig-Penney model of 1-dimensional solids (under the approximation of periodic energy barriers that are delta functions) requires that $k$ be restricted to values that makes the following relationship true:

$$\mu \left( \frac{\sin \alpha a}{\alpha a} \right) + \cos \alpha a = \cos ka$$

where $\mu$ is the barrier strength, $a$ is the interatomic distance and $\alpha$ is related to energy.

   (a) Plot the left hand side of this relationship as a function of $\alpha a$ for an arbitrary value of $\mu$ and indicate the regions of forbidden values of $k$. Explain why these are forbidden.

   (b) Determine the width of the first energy gap predicted by this model in terms of $\mu$ and $a$. In order to obtain an analytical answer, expand $\sin(x)$ and $\cos(x)$ to their algebraic forms, using terms no higher than $x^2$.

4. Consider a two-dimensional solid comprised of A atoms distributed on a square lattice. The lattice constant is $a$. The Bloch sum wave function is:

$$\psi_k(\vec{r}_{p,q}) = \sum_{n=1}^{N} e^{i k \cdot \vec{r}_{p,q}} \chi_{p,q}(r) = \sum_{n=1}^{N} e^{i (k, p, a + k, q, a)} \chi_{p,q}(r)$$

   (a) Plot the phases at the atoms [the value of $\cos(k \cdot \vec{r}_{p,q})$] for $k = (0, 0)$, $(\pi/a, 0)$, $(0, \pi/a)$ and $(\pi/a, \pi/a)$.

   (b) Plot energy as a function of $k_x$, $k_y$ in the form of a contour plot over the first Brillouin zone, $(-\pi/a, -\pi/a)$ to $(\pi/a, \pi/a)$, for the s orbitals. Indicate the value of $E(k)$ at $(0, 0)$, $(\pi/a, 0)$, and $(\pi/a, \pi/a)$

   (c) Plot $E(k)$ in a standard x-y plot, where $k$ follows the triangular path from $(0,0) = \Gamma$, to $(\pi/a, 0) = X$, to $(\pi/a, \pi/a) = M$, and back to $(0,0) = \Gamma$.

   (d) Make a sketch of $p_x$ and $p_y$ orbitals (with the phases on the lobes indicated) for $k = (0, 0)$, $(\pi/a, 0)$, $(0, \pi/a)$ and $(\pi/a, \pi/a)$. Indicate the kind of bonding or anti-bonding that these states represent ($\sigma$ and/or $\pi$). Rank the energies for these states.